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The non-abelian open superstring effective action through order α'^3

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Abstract: Using the method developed in `hep-th/0103015`, we determine the non-abelian Born-Infeld action through $\mathcal{O}(\alpha'^3)$. We start from solutions to a Yang-Mills theory which define a stable holomorphic vector bundle. In D-brane context this corresponds to BPS configurations in the limit of small background fields. Subsequently we investigate its deformation away from this limit. Through $\mathcal{O}(\alpha'^2)$, a unique, modulo field redefinitions, solution emerges. At $\mathcal{O}(\alpha'^3)$ we find a one-parameter family of allowed deformations. The presence of derivative terms turns out to be essential. Finally, we present a detailed comparison of our results to existing, partial results.

Keywords: Solitons Monopoles and Instantons, D-branes.

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Contents

1. Introduction

The bosonic massless degrees of freedom of an open string ending on a flat Dp-brane are a U(1) gauge field, associated to excitations of the string longitudinal to the brane, and neutral scalar fields, describing the fluctuations of the brane in the transverse directions [1]. For slowly varying fields, i.e. ignoring derivative terms, the effective action for these massless degrees of freedom is known through all orders in α'. It is the $d = 10$ supersymmetric abelian Born-Infeld theory, dimensionally reduced to $p + 1$ dimensions [2, 3].

Once several, say n, D-branes coincide, the gauge group gets enhanced from U(1) to U(n), [4]. In leading order in α', the effective action is precisely the $d = 10$ supersymmetric U(n) Yang-Mills theory dimensionally reduced to $p + 1$ dimensions. The exact structure of the full effective action remains an elusive puzzle. Two complications arise in comparison with the abelian case:

- Because all fields transform in the adjoint of U(n), an ordering prescription is needed.

- There is no covariant notion of a slowly varying field. In other words, higher order derivative terms have to be included.

The calculation of open superstring amplitudes allows for a direct determination of the effective action. This approach lead to firm results through order α'^2 [5, 6, 7]. A first systematic investigation of the effective action was performed in [8]. A first observation is that, as the effective action has to match gluon disk amplitudes, there is necessarily only a single overall group trace. Furthermore, the effective action $\mathcal{S}$ is necessarily of the form $\mathcal{S} = \mathcal{S}_1 + \mathcal{S}_2 + \mathcal{S}_3$. Here, $\mathcal{S}_1$ does not contain any covariant derivatives acting on the fieldstrength and is, by definition, the non-abelian Born-Infeld action. Both $\mathcal{S}_2$ and $\mathcal{S}_3$ contain the terms with derivatives acting on the fieldstrength, but while $\mathcal{S}_2$ has only terms with symmetrized products of covariant derivatives, $\mathcal{S}_3$ has anti-symmetrized products of covariant derivatives as well. Because of $[D, D]\cdot = [F, \cdot]$, the separation between $\mathcal{S}_1$ and $\mathcal{S}_3$ is not unambiguously defined. This ambiguity was fixed in [8] by proposing that $\mathcal{S}_1$ is the non-abelian Born-Infeld action defined by means of the symmetrized trace prescription. It assumes the same form as the abelian Born-Infeld action but upon expanding the action in powers of the fieldstrength, one first symmetrizes all terms and subsequently one performs the group trace. Indeed, all other terms without derivatives not belonging to this class can be rewritten as elements of $\mathcal{S}_3$. In the abelian limit, $\mathcal{S}_1$ reduces then to the abelian Born-Infeld action, $\mathcal{S}_3$ vanishes and $\mathcal{S}_2$, which is present [9], vanishes in the limit of slowly varying fields. Through order α'^2, $\mathcal{S}_2$ and $\mathcal{S}_3$ vanish. At higher orders, contributions to those terms are expected as well. Indeed, the results in [10] and [11] demonstrated that knowledge of $\mathcal{S}_1$ is not sufficient to reproduce even simple features of D-brane dynamics. As we will show in this paper, from order α'^3 on, $\mathcal{S}_2$ and $\mathcal{S}_3$ receive non-trivial contributions as well.

A direct calculation at higher orders becomes technically very involved (however see [12] and [13] for partial results in this direction), so one is forced to develop alternative approaches. One of these, motivated by the results in [10] and [11], uses the mass spectrum as a guideline which resulted in partial higher order results through order α'^4 [14].

The problem at hand possesses a lot of supersymmetry: there are 16 linearly and 16 non-linearly realized supersymmetries. This rises the hope that requiring the deformations of the Yang-Mills action to be supersymmetric will severely restrict the possibilities, perhaps even leading to a unique all order result [15, 16]. Recently it was shown at component level that supersymmetry indeed almost fully determines the action through $\mathcal{O}(\alpha'^2)$ including fermionic and derivative terms [17]. The presence of both linear and non-linear supersymmetry would suggest the existence of an explicitly κ-invariant formulation. Despite an attempt at lower order [18], this remains an open problem [7].

A related approach was recently developed in [19]. Starting from the $N = 4$, $d = 4$ supersymmetric Yang-Mills theory, the bosonic part of the one-loop effective action through operators of dimension 10 was calculated. If one assumes that the supersymmetric deformation of the Yang-Mills action is unique, then this calculation should yield the non-abelian open superstring action. However, as we will discuss in section 6, this is not so. In addition, the method of [19] is restricted to four dimensions.

A very different approach was launched in [20]. Starting point was the existence of a particular class of solutions to Yang-Mills generalizing the usual instantons in four dimensions. These solutions define stable holomorphic bundles [21]. In the context of D-brane physics, such solutions correspond to BPS solutions in the weak field limit [22]. In [20], deformations of such solutions were analyzed in the abelian limit. Arbitrary powers of the fieldstrength were added to the Yang-Mills action. Subsequently it was required that stable holomorphic bundles, or some deformation thereof, still provides solutions to the equations of motion. Surprisingly this approach leads to a *unique* deformation: the abelian Born-Infeld action. While the holomorphicity condition remains unchanged, the stability condition acquires higher order corrections as well.

An obvious question which arises in this context is whether the method sketched above leads to similar restrictions in the non-abelian case. The analysis in [20] used the explicit assumption that only slowly varying fields appeared. In other words, the Yang-Mills action was deformed by adding arbitrary powers of the fieldstrength to it, but terms containing derivatives of the fieldstrength were excluded. In the non-abelian case, there is no covariant notion of acceleration terms. Indeed, in the abelian case, it is not hard to find a rescaling of the coordinates and the gauge fields such that a limit exists where the fieldstrength remains invariant but its derivatives vanish. Such a limit does not exist in the non-abelian case. However, one might still try to repeat the analysis of [20] under the same assumptions. When doing this, we found no contribution at $\mathcal{O}(\alpha'^3)$, but we encountered an obstruction at order α'^4 (i.e. order F^6). This clearly showed the need to include derivative terms as well in the non-abelian case. For alternative arguments we refer to the introduction of [13] (see also [12] and [19]).

As a first test, we analyze the deformation of the non-abelian Yang-Mills action through order α'^3. At this order partial results were obtained before [12], [13], [19]. A clear full answer is however still lacking. The present method shows a major drawback once derivative terms are allowed: the number of terms which can potentially contribute to the action increases dramatically with each order in α'. An additional difficulty is that, because of partial integration, Bianchi identities and the $[D, D]\cdot = [F, \cdot]$ identity, many relations between various terms exist. In order to deal with this in an efficient way, we wrote a program [23] in Java, an object oriented language based on the syntax of C, which classifies at a given order in α' the indepen-

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dent terms in the action, calculates the resulting equations of motion and analyzes the deformations of the stability condition. Finally it takes care of field redefinitions as well.

This paper is organized as follows. In the next section 2 we briefly review the relevant solutions in Yang-Mills. In sections 3 to 5 we study the deformations through order α'^3 and systematize our method. In the last section 6 we analyze our results and confront them with previously known results. The first appendix explains our conventions and notations. Appendix B gives the equations of motions at order α'^2 and appendix C a base for the lagrangian and the stability condition deformation at order α'^3.

2. The leading order: stable holomorphic bundles in Yang-Mills

In leading order, the effective action of n coinciding Dp-branes is the supersymmetric U(n) Yang-Mills theory in ten dimensions,[1]

$$\mathcal{S} = \int d^{10}x \,\mathrm{tr}\, \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \bar{\psi} \not{D} \psi \right), \tag{2.1}$$

dimensionally reduced to $p+1$ dimensions. The Majorana-Weyl spinor ψ transforms in the adjoint representation of U(n). In this way we get as world-volume degrees of freedom a U(n) gauge field in $p+1$ dimensions, $9-p$ scalar fields and the 16 components of ψ.

In the present paper, we will ignore the transversal scalars and the fermionic degrees of freedom as they do not seem to give additional information. Furthermore, we make one important assumption: instead of restricting ourselves to $d = 10$ or less, we will require our analysis to hold in any even dimension! In this way we avoid relations which exist in particular dimensions.

Our starting point is a U(n) Yang-Mills action in an even dimensional, flat euclidean space,[2]

$$\mathcal{L}_{(0)} = \frac{1}{4} \mathrm{tr}\, F_{\mu_1\mu_2} F_{\mu_2\mu_1} \,. \tag{2.2}$$

In complex coordinates, the equations of motion, $D_\nu F_{\nu\mu} = 0$, read,

$$\begin{aligned} 0 &= D_{\bar{\alpha}} F_{\alpha\bar{\beta}} + D_\alpha F_{\bar{\alpha}\bar{\beta}} \\ &= D_{\bar{\beta}} F_{\alpha\bar{\alpha}} + 2 D_\alpha F_{\bar{\alpha}\bar{\beta}} \,, \end{aligned} \tag{2.3}$$

[1] We ignore an overall multiplicative constant.

[2] As the metric is $+1$ in all directions, we simplify the notation by putting all indices down. Unless stated otherwise, we sum over repeated indices. Furthermore, the lagrangian eq. (2.2), should still be multiplied by an arbitrary coupling constant $-1/g^2$.

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where we used the Bianchi identities in the last line. One sees that configurations satisfying

$$F_{\alpha\beta} = F_{\bar{\alpha}\bar{\beta}} = 0, \tag{2.4}$$

and

$$g^{\alpha\bar{\beta}} F_{\alpha\bar{\beta}} = \sum_{\alpha} F_{\alpha\bar{\alpha}} \equiv F_{\alpha\bar{\alpha}} = 0\,, \tag{2.5}$$

solve the equations of motion [21]. Eq. (2.4) defines a holomorphic bundle, while eq. (2.5) guarantees the stability of the bundle [24]. We will alternatively call the latter equation the Donaldson-Uhlenbeck-Yau condition, henceforth abbreviated DUY condition.

Restricting to dimensions less than 10, these solutions are BPS configurations of D-branes in the limit where $2\pi\alpha' F$ is small. When $p = 2$, the BPS conditions are recognized as the standard instanton equations. Note that constant magnetic background fields which satisfy the conditions eqs. (2.4) and (2.5) can be reinterpreted, after T-dualization, as BPS configurations of Dp-branes at angles [25].

In the next sections, we will investigate order by order in α' the most general deformation of eq. (2.2). So we will add at each order the most general polynomial in the fieldstrength and its covariant derivatives, each term with an arbitrary coupling constant. Subsequently we will demand that configurations of the form eqs. (2.4) and (2.5) solve the deformed equations of motion. As it turns out this will fix, modulo certain field redefinitions, the coupling constants. Simultaneously, we will have to deform the stability condition eq. (2.5) as well. Concerning the deformation of eq. (2.5), we require it to be such that it fully determines $F_{\alpha\bar{\alpha}}$ i.e. *it should be such that $F_{\alpha\bar{\alpha}}$ only appears at the left-hand side of the equation.*

3. Learning from the low orders

3.1 The α'^1 corrections

The most general deformation of the Yang-Mills action, $\mathcal{L}_{(0)}$, at the next order is given by $\mathcal{L}_{(0)} + \mathcal{L}_{(1)}$ with,

$$\mathcal{L}_{(1)} = 2\pi\alpha' l^3_{0,0,0} \mathrm{tr}\, \left(F_{\mu_1\mu_2} F_{\mu_2\mu_3} F_{\mu_3\mu_1}\right) + 2\pi\alpha' l^3_{1,1,0} \mathrm{tr}\, \left(\left(D_{\mu_3} D_{\mu_1} F_{\mu_1\mu_2}\right) F_{\mu_2\mu_3}\right)\,, \tag{3.1}$$

with $l^3_{0,0,0}$ and $l^3_{1,1,0}$ arbitrary constants. This expression takes into account partial integration, Bianchi and $[D, D]\cdot = [F, \cdot]$ identities, as we will study in more detail in section 4. In order to simplify our notation, we will put $2\pi\alpha' = 1$ from now on. In complex coordinates, one finds that the equations of motion following from eq. (3.1) read as

$$
\begin{aligned}
0 = {} & D_{\bar\beta}F_{\alpha\bar\alpha} - 3\, l^3_{0,0,0} \left(D_{\bar\beta}F_{\alpha_1\bar\alpha_2}\right) F_{\alpha_2\bar\alpha_1} + 3\, l^3_{0,0,0} F_{\alpha_1\bar\alpha_2} \left(D_{\bar\beta}F_{\alpha_2\bar\alpha_1}\right) + \\
& +2\, l^3_{1,1,0} \left(D_{\bar\beta}F_{\alpha_1\bar\alpha_1}\right) F_{\alpha_2\bar\alpha_2} - 2\, l^3_{1,1,0} F_{\alpha_1\bar\alpha_1} \left(D_{\bar\beta}F_{\alpha_2\bar\alpha_2}\right) + \\
& + \left(4\, l^3_{1,1,0} + 3\, l^3_{0,0,0}\right) \left(D_{\bar\beta}D_{\bar\alpha_1}D_{\alpha_1}F_{\alpha_2\bar\alpha_2}\right) - 3\, l^3_{0,0,0} \left(D_{\alpha_1}D_{\bar\beta}D_{\bar\alpha_1}F_{\alpha_2\bar\alpha_2}\right), \quad (3.2)
\end{aligned}
$$

where we used eq. (2.4) and the Bianchi identities. Almost all terms vanish when implementing eq. (2.5), leaving only the second and third term. At this order the only allowed deformation[3] of eq. (2.5) is,

$$
0 = F_{\alpha\bar\alpha} + d^3_{0,0,0} F_{\alpha_1\bar\alpha_2} F_{\alpha_2\bar\alpha_1} \,. \quad (3.3)
$$

One sees immediately that the equations of motion are solved provided we do not deform eq. (2.5), i.e. $d^3_{0,0,0} = 0$ and we put $l^3_{0,0,0} = 0$. This eliminates the second and the third term in eq. (3.2). The remainder of the equation of motion is satisfied by virtue of eq. (2.5). The surviving second term in $\mathcal{L}_{(1)}$ can then be eliminated by a field redefinition,

$$
A_\mu \longrightarrow A_\mu - l^3_{1,1,0} D_\nu F_{\nu\mu}, \quad (3.4)
$$

which exhausts the field redefinitions at this order. Concluding the $\mathcal{O}(\alpha')$ deformation of both the Yang-Mills action and the stability condition vanish, which is consistent with direct calculations.

3.2 The α'^2 corrections

At the next order the most general deformation of the Yang-Mills lagrangian reads as

$$
\mathcal{L} = \mathcal{L}_{(0)} + \mathcal{L}_{(2)} \,, \quad (3.5)
$$

where $\mathcal{L}_{(0)}$ is given in eq. (2.2) and $\mathcal{L}_{(2)}$ is

$$
\begin{aligned}
\mathcal{L}_{(2)} = \operatorname{tr} \Big(& l^4_{0,0,0} F_{\mu_1\mu_2}F_{\mu_2\mu_3}F_{\mu_3\mu_4}F_{\mu_4\mu_1} + l^4_{0,0,1} F_{\mu_1\mu_2}F_{\mu_2\mu_3}F_{\mu_4\mu_1}F_{\mu_3\mu_4} + \\
& + l^4_{0,1,0} F_{\mu_1\mu_2}F_{\mu_2\mu_1}F_{\mu_3\mu_4}F_{\mu_4\mu_3} + l^4_{0,1,1} F_{\mu_1\mu_2}F_{\mu_3\mu_4}F_{\mu_2\mu_1}F_{\mu_4\mu_3} + \\
& + l^4_{1,2,1} \left(D_{\mu_4}D_{\mu_1}F_{\mu_1\mu_2}\right) F_{\mu_3\mu_4}F_{\mu_2\mu_3} + l^4_{1,2,6} \left(D_{\mu_1}F_{\mu_1\mu_2}\right) F_{\mu_2\mu_3} \left(D_{\mu_4}F_{\mu_3\mu_4}\right) + \\
& + l^4_{2,1,23} \left(D_{\mu_4}D_{\mu_1}F_{\mu_1\mu_2}\right) \left(D_{\mu_4}D_{\mu_3}F_{\mu_2\mu_3}\right) \Big) \,. \quad (3.6)
\end{aligned}
$$

This is the most general deformation at order α'^2 where we used the Bianchi, partial integration and $[D,D]\cdot = [F,\cdot]$ identities. Both the deformation of the stability condition, eq. (2.5) and the contribution to the equations of motion at this order are explicitly given in appendix B.

[3]We remind the reader that we view the deformed stability condition as an expression for $F_{\alpha\bar\alpha}$. So while additional deformation terms of the form $F_{\alpha_1\bar\alpha_1}F_{\alpha_2\bar\alpha_2}$ or $(D_{\bar\alpha_1}D_{\alpha_1}F_{\alpha_2\bar\alpha_2})$ are dimensionally allowed, they are excluded by our ansatz.

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As an illustration, we analyze the equations of motion in some detail. It is clear that the equations of the type $e^4_{0,0,s_3}$ and $e^4_{0,3,s_3}$ are satisfied provided

$$
\begin{aligned}
& d^4_{0,0,0} = d^4_{0,0,1} = 4l^4_{0,0,0} = 2l^4_{0,0,1} = -8l^4_{0,1,0} = -16l^4_{0,1,1}\,, \\
& l^4_{1,2,1} = 0\,.
\end{aligned} \tag{3.7}
$$

The contributions of the type $e^4_{1,0,s_3}$ vanish, provided

$$
d^4_{1,0,4} = d^4_{1,0,5} = 0\,, \tag{3.8}
$$

holds. The remainder of the equations of motion now trivially vanishes when applying eq. (2.5).[4]

We can fix one more parameter. Initially we had two choices: we could choose the overall multiplicative constant in front of the action and we can fix the scale of the fieldstrength F. These two arbitrary constants are fixed by choosing the conventional factor 1/4 in front of the leading term in the action and by putting[5] $l^4_{0,0,0} = -1/24$.

Having done this we fixed the deformation of the action completely modulo the coupling constants $l^4_{1,2,6}$ and $l^4_{2,1,23}$. However, we still have to consider field redefinitions. The most general field redefinition relevant at this order is

$$
\begin{aligned}
A_\nu \to A_\nu &+ f^3_{0,1,0}\left(D_{\mu_1}F_{\mu_1\mu_2}\right)F_{\mu_2\nu} + f^3_{0,1,1}F_{\mu_2\nu}\left(D_{\mu_1}F_{\mu_1\mu_2}\right) + \\
&+ f^3_{0,1,2}F_{\mu_1\mu_2}\left(D_{\mu_1}F_{\mu_2\nu}\right) + f^3_{0,1,3}\left(D_{\mu_1}F_{\mu_2\nu}\right)F_{\mu_1\mu_2} + \\
&+ f^3_{1,0,1}\left(D_{\mu_1}D_{\mu_2}D_{\mu_1}F_{\mu_2\nu}\right),
\end{aligned} \tag{3.9}
$$

where again we took Bianchi identities and the $[D,D]\cdot = [F,\cdot]$ relation into account. The coupling constants transform as follows under the field redefinitions,

$$
\begin{aligned}
l^4_{0,s_2,s_3} &\to l^4_{0,s_2,s_3}\,, \\
l^4_{1,2,1} &\to l^4_{1,2,1} - f^3_{0,1,2} + 2f^3_{1,0,1} + f^3_{0,1,3}\,, \\
l^4_{1,2,6} &\to l^4_{1,2,6} + f^3_{0,1,1} + f^3_{0,1,2} - f^3_{0,1,0} - f^3_{0,1,3}\,, \\
l^4_{2,1,23} &\to l^4_{2,1,23} + f^3_{1,0,1}\,.
\end{aligned} \tag{3.10}
$$

Taking into account that we have to keep $l^4_{1,2,1} = 0$, we find that the three field redefinitions

$$
\begin{aligned}
f^3_{0,1,1} - f^3_{0,1,0} &= -l^4_{1,2,6} + 2l^4_{2,1,23}\,, \\
f^3_{0,1,2} - f^3_{0,1,3} &= -2l^4_{2,1,23}\,, \\
f^3_{1,0,1} &= -l^4_{2,1,23}\,,
\end{aligned} \tag{3.11}
$$

[4] Of course these terms will contribute at order α'^4 as a consequence of the deformation eq. (B.2). See section 4, step 7.b and 7.c for a systematic approach.

[5] The unconventional $-$ sign is due to the fact that we choose an anti-hermitean basis for $u(n)$.

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eliminate the derivative terms in eq. (3.6). This leaves us two field redefinitions,

$$\begin{aligned} A_\nu \to A_\nu + \frac{1}{2}\left(f^3_{0,1,0} + f^3_{0,1,1}\right)\left\{\left(D_{\mu_1}F_{\mu_1\mu_2}\right), F_{\mu_2\nu}\right\} + \\ +\frac{1}{2}\left(f^3_{0,1,2} + f^3_{0,1,3}\right)\left\{F_{\mu_1\mu_2}, \left(D_{\mu_1}F_{\mu_2\nu}\right)\right\}, \end{aligned} \tag{3.12}$$

which will not play any further role in this paper as they only become potentially relevant at order α'^4 in the action. Note that it is quite remarkable that certain terms which are removable through field redefinitions, the $l^4_{1,2,1}$ term in casu, can get fixed by our method.

Summarizing, through order α'^2, the lagrangian is given by $\mathcal{L} = \mathcal{L}_{(0)} + \mathcal{L}_{(2)}$, with $\mathcal{L}_{(0)}$ given in eq. (2.2) and

$$\begin{aligned} \mathcal{L}_{(2)} = -\mathrm{tr}\Bigg(&\frac{1}{24}F_{\mu_1\mu_2}F_{\mu_2\mu_3}F_{\mu_3\mu_4}F_{\mu_4\mu_1} + \frac{1}{12}F_{\mu_1\mu_2}F_{\mu_2\mu_3}F_{\mu_4\mu_1}F_{\mu_3\mu_4} - \\ &- \frac{1}{48}F_{\mu_1\mu_2}F_{\mu_2\mu_1}F_{\mu_3\mu_4}F_{\mu_4\mu_3} - \frac{1}{96}F_{\mu_1\mu_2}F_{\mu_3\mu_4}F_{\mu_2\mu_1}F_{\mu_4\mu_3}\Bigg). \end{aligned} \tag{3.13}$$

Configurations satisfying eq. (2.4) and

$$0 = F_{\alpha\bar\alpha} - \frac{1}{6}F_{\alpha_1\bar\alpha_2}F_{\alpha_2\bar\alpha_3}F_{\alpha_3\bar\alpha_1} - \frac{1}{6}F_{\alpha_1\bar\alpha_2}F_{\alpha_3\bar\alpha_1}F_{\alpha_2\bar\alpha_3}, \tag{3.14}$$

solve the equations of motion. This result is again fully consistent with direct calculations.

4. Systematizing our method

After studying the low order cases, we are ready to put together the calculational scheme. Although this scheme can be slavishly followed at higher orders, the calculations itself will become extremely lengthy. So they were carried out by a computer program, written specially for the task at hand. The language of choice was Java, which as a modern object oriented programming language proved to us more user-friendly than the in physics more commonly used C or Fortran. In this section we will discuss what it does, and leave the implementational details for what they are. More details as well as the source code will be given in [23].

Roughly put, the program will construct the most general lagrangian and deformation of the DUY condition at each order in α'. Subsequently, we impose that fieldstrength configurations satisfying eq. (2.4) and the generalized DUY condition solve the equations of motion. This generates a set of equations, since the coefficient of each *independent* term in the equations of motion has to be zero. From these conditions we can fix the coefficients of the lagrangian *as well* as the coefficients of the DUY deformation. Having said this, we can delve into the technical details.

Properties	Lagrangian	Field redefinitions	DUY deformation	Equations of motion
Group trace implied[a]	yes	no	no	no
Free index	no	yes	no	yes
Complex coordinates[b]	no	no	yes	yes
Type of identities	Bianchi PI[c], DDF[d]	Bianchi DDF[d]	Bianchi DDF[d]	Bianchi DDF[d]
Used Symbols[e]	$\mathcal{L}$ l	$\mathcal{F}$ f	$\mathcal{D}$ d	$\mathcal{E}$ e

[a]So there is cyclic symmetry

[b]See appendix A.

[c]PI: partial integration identities.

[d]DDF: identities of the type $[D, D]\cdot = [F, \cdot]$.

[e]The curly type is used for denoting all terms at a certain order, the small type in the labelling of the individual terms (as explained in appendix A).

Table 1: Properties of the different types of terms.

The program distinguishes 4 kinds of terms at each order: their properties are listed in table 1. For each of these types, the program will have to:

1. Calculate all possible terms, using as building blocks antisymmetric fields F and covariant derivatives. These terms have a priori arbitrary coefficients, which are labelled according to the classification scheme of appendix A.

2. Calculate all possible identities between those terms: these are the partial integration identities (only for the lagrangian), the Bianchi identities and the identities of the type $[D, D]\cdot = [F, \cdot]$.

3. Solve those (linear) identities and thus separate the linearly dependent terms from the linearly independent terms, forming a base. The program proceeds by eliminating one term out of each equation. Of course there is still the freedom of choosing the term to eliminate and this will often give rise to some priority rule.

   Sometimes a term will carry some coefficient information. Upon elimination, this information will be transferred to the other terms in the equation. An example: suppose $T^1, T^2, \ldots T^n$ are terms in an equation of motion, which reads:

$$\sum_{j=1}^{n} c_j T^j = 0\,. \tag{4.1}$$

Term T^j is said to *carry* its coefficient c_j. Now, if T^i is eliminated from the identity:

$$T^i = \sum_{j\neq i} d_j T^j \,, \tag{4.2}$$

it has to transfer his coefficient c_i to the other terms and the equation of motion becomes:

$$\sum_{j\neq i} (c_j + d_j c_i) T^j = 0 \,. \tag{4.3}$$

Because of the arbitrariness in choosing a base, many lagrangians will in fact be equivalent. When comparing with the results in the literature [19, 12], we will have to express their terms in our base. Also, as an extra complication, we have to take account of the fact that the coefficients of some terms in the lagrangian may change when applying a field redefinition and allow for them to differ [6]. We call the latter from now on *FR changeable terms*.

After these initial remarks we state the algorithm, which has to be repeated order per order[6] in α':

1. Construct all possible terms in the lagrangian and all identities among those.

2. If we want to know which terms are FR changeable, we must put in quite a lot of extra work. To the solution for the lagrangian already found at lower orders, apply a field redefinition:

$$A_\mu \to A_\mu + \mathcal{F}_\mu \,, \tag{4.4}$$

where $\mathcal{F}_\mu$ is a linear combination of all possible *independent* field redefinition terms of the appropriate lower orders. Observe how the coefficients of the terms in the lagrangian at the present order change. To clarify what we mean, we study the simplest case in detail. At order α'^0, the most general field redefinition is:

$$\mathcal{F}_{(0),\nu} = f^2_{0,0,0} D_{\mu_1} F_{\mu_1\nu} \,, \tag{4.5}$$

and, as we have already used in section 3.1, the coefficient change of term $l^3_{1,1,0}$ (see (3.1)) becomes

$$\Delta l^3_{1,1,0} = f^2_{0,0,0} \,. \tag{4.6}$$

[6]We will call the order in F n, so the order in α' is $n-2$.

For each FR changeable term, we say that the term *carries*[7] his coefficient change. The other terms, we will call *empty* i.e. carrying nothing.

3. Calculate all independent terms in the lagrangian. When eliminating an FR changeable term out of a certain equation, its coefficient change must be transferred as explained, because we want to know how the remaining terms transform under a field redefinition. So all the other terms in that equation become FR changeable. To minimize the amount of FR changeable terms we end up with, we use the following priority rule: eliminate empty terms first. If we did not follow this rule the FR changeability would quickly spread among all independent terms, although many coefficient changes would be dependent.[8]

   In the end, when comparing the result to the literature, we will only have to consider empty terms.

4. Construct all possible *independent* field redefinition terms for later use at higher orders.

5. Construct all possible *independent* contributions to the DUY condition. As an example, $\mathcal{D}_{(2)}$ from eq. (B.2) would be the result at order α'^2.

6. Construct all possible terms in the equations of motion.

7. The coefficients of those terms in the equations of motions will have three contributions:

   (a) The coefficients obtained from varying the terms in the lagrangian containing the arbitrary lagrangian coefficients $l^n_{s_1,s_2,s_3}$. Note that in the non-abelian case there is also a contribution from varying the covariant derivatives.

   (b) Subtraction of the deformed DUY condition:

   $$D_{\bar\beta}(F_{\alpha_1\bar\alpha_1} + \cdots + \mathcal{D}_{(n-3)} + \mathcal{D}_{(n-2)}) = 0\,. \tag{4.7}$$

   The first term cancels the contribution of the lagrangian to the equations of motion at order α'^0. The last term $\mathcal{D}_{(n-2)}$ contributes to the equations of motion at the present order $\alpha'^{(n-2)}$.

[7] An alternative way of looking at this, is that the field redefinition terms form a dual vector space, because the choice of a lagrangian term and a field redefinition term produces a (fractional) number, i.e. the coefficient change. More precisely, the field redefinition terms are only a subspace of the dual vector space and we want to make this clear by an appropriate choice of base of the original space.

[8] Note that our "rule of thumb" doesn't guarantee that *all* remaining coefficient changes are independent, but certainly most of them.

Order	Maximum number of 1-loops
α'	1
α'^2	1
α'^3	0
α'^4	0

Table 2: Terms with more 1-loops than indicated are not considered.

(c) Consider a term with one or more 1-loops:

$$F_{\alpha_1\bar{\alpha}_1}\ldots F_{\alpha_i\bar{\alpha}_i}\ldots F_{\alpha_p\bar{\alpha}_p}(\mathrm{tail})\,. \tag{4.8}$$

We can manipulate its coefficient, because we can add a "derived" DUY condition:

$$o_i\, F_{\alpha_1\bar{\alpha}_1}\ldots(F_{\alpha_i\bar{\alpha}_i}+\mathcal{D}_{(1)}+\mathcal{D}_{(2)}+\cdots)\ldots F_{\alpha_p\bar{\alpha}_p}(\mathrm{tail})=0\,, \tag{4.9}$$

for each 1-loop, which will contribute a factor o_i to the term under consideration. This o_i can be considered as an extra degree of freedom, which can be adjusted to make the coefficient of the 1-loop term in the equations of motion zero. Obviously this adjustment has implications at higher orders. If we started from a term with p 1-loops, each term in the deformed DUY condition at higher order contains $p-1+r$ 1-loops, where r is the number of 1-loops in the DUY term. So if $p\neq 1$ or $r\neq 0$, we still have a term with 1-loops and can absorb the extra factor o_i by again adding a new "derived" DUY condition.

Therefore we only have to be careful if we eventually end up with a term without 1-loops. As we have seen in section 3.1, $d^3_{0,0,0}=0$, so at order α' there are no DUY terms without 1-loops ($r\neq 0$). So if we want to lower the number of 1-loops by substituting a DUY condition, we end up at least 2 orders of α' higher. Since the highest order we hope to study is α'^4, at that order we do not have to consider 1-loop terms. Also at order α'^3, we do not have to consider 1-loop terms, because they will only influence terms without 1-loops for the first time at order α'^5 — two orders higher. Continuing this reasoning, we obtain table 2.

Concluding, we never consider terms with more than one 1-loop, so there is only one extra unknown per 1-loop term, which we will call $o^n_{s_1,s_2,s_3}$ after that 1-loop term.

As a next step, we want to get rid of the o-coefficients altogether. The reader can convince himself that, provided one uses the priority rule described in the next paragraph, there is no loss of generality if one only introduces the o-factors after the elimination process, instead of before.[9] But now things get very easy because every independent 1-loop term will have one and only one o-factor, that will be adjusted to get the coefficient zero. In appendix B the o-coefficients are implicit.

8. Calculate the independent terms in the equations of motion. When eliminating a term from an equation, its coefficient has to be appropriately transferred to the other terms in the equation. The priority rule reads: always try to keep terms with as many 1-loops as possible. This means that a term will never transfer its coefficient to a term with less 1-loops. So, if we do not have to consider the coefficients of terms with a certain number of 1-loops at the end, we might as well not construct them in the first place and leave them out throughout the whole calculation.

9. Because we are searching for solutions, the equations of motion have to be zero. Since we now have independent terms, their coefficients separately have to be zero. From this set of equations, we can solve for the unknowns $l^n_{s_1,s_2,s_3}$ and $d^n_{s_1,s_2,s_3}$.

5. Uncharted territory: the α'^3 corrections

Pushing our method to order α'^3, we enter largely uncharted territory. At the same time, our approach enters a new level of complexity: the most general deformation of the lagrangian at this order consists of no less than 36 terms while the most general deformation of the stability condition at this order, consistent with our assumptions, counts 27 terms. Using the same strategy as at lower orders, but now largely relying on our program, we arrive at the following result (see appendix C for the terms themselves):

$$\begin{aligned} -l^5_{0,0,1} &= -l^5_{0,0,3} = 2\, l^5_{1,0,4} = 2\, l^5_{1,0,6} = -8 l^5_{1,1,4} = l^5_{1,4,30} = -l^5_{1,4,58} = 2\, l^5_{0,1,1} = \lambda \\ l^5_{0,0,0} &= l^5_{0,0,2} = l^5_{0,1,0} = l^5_{1,1,3} = l^5_{1,4,47} = 0\,, \end{aligned} \tag{5.1}$$

with

$$\lambda = d^5_{0,0,4} + d^5_{1,0,12}\,. \tag{5.2}$$

[9]Basically, this is permissible because the identities that one uses to eliminate the dependent 1-loop terms, still apply when the 1-loop is replaced by a higher order DUY deformation piece. So at that higher order the o-coefficients will hook up together in the same groups as in the coefficients of the independent 1-loop terms at lower order

As for the DUY deformation:

$$\begin{aligned} d^5_{0,0,2} &= -d^5_{0,0,4} \\ d^5_{0,0,1} &= d^5_{0,0,3} = d^5_{0,1,x} = d^5_{1,1,x} = d^5_{2,0,x} = 0 \\ -2\, d^5_{0,0,0} &= 2\, d^5_{0,0,5} = d^5_{0,0,4} + 7\, d^5_{1,0,12} \\ 2\, d^5_{1,0,13} &= 2d^5_{1,0,16} = 2d^5_{1,0,19} = 2d^5_{1,0,22} = d^5_{0,0,4} + 3d^5_{1,0,12} \\ d^5_{1,0,14} &= d^5_{1,0,15} = d^5_{1,0,17} = d^5_{1,0,18} = d^5_{1,0,20} = d^5_{1,0,21} = d^5_{1,0,23} = d^5_{1,0,12}\,. \end{aligned} \tag{5.3}$$

It turns out that 23 terms in the deformation of the lagrangian can be removed through field redefinitions, so we did not list them in eq. (5.1). The terms which are left are insensitive to field redefinitions. Note that the solution for the lagrangian is unique up to a multiplicative factor; the λ in eq. (5.2) can be freely chosen by juggling with $d^5_{0,0,4}$ and/or $d^5_{1,0,12}$ in the DUY condition.

Extending our assumption about the 1-loops in the DUY condition, all terms with "subloops" turn out to be vanishing.

6. Discussion and conclusions

In this paper we started from solutions to Yang-Mills which define a stable holomorphic bundle, eqs. (2.4) and (2.5). In the context of D-brane physics this corresponds to BPS configurations in the limit where the magnetic background fields are small. Subsequently we investigated the deformations of both the action and the stability condition. As deformations we allowed for arbitrary, independent powers of the fieldstrength and covariant derivatives thereof. Requiring that these configurations solve the (deformed) equations of motion largely fixes the allowed deformations.

Novel compared to the analysis in [20], is the necessity to include terms in the action with derivatives acting on the fieldstrengths. Indeed an initial attempt which considered only deformations polynomial in the fieldstrength failed at order α'^4!

In the present paper we studied the deformation through order α'^3. Through order α'^2 we reproduce the well-known results, eqs. (2.2) and (3.13). The stability condition gets deformed as in eq. (3.14).

In the previous section we pushed our method to the next order. At order α'^3 we found a one parameter family of allowed deformations,

$$\begin{aligned} \mathcal{L}_{(3)} = -\lambda\, \mathrm{tr}\, \Big(& F_{\mu_1\mu_2}F_{\mu_2\mu_3}F_{\mu_3\mu_4}F_{\mu_5\mu_1}F_{\mu_4\mu_5} + F_{\mu_1\mu_2}F_{\mu_4\mu_5}F_{\mu_2\mu_3}F_{\mu_5\mu_1}F_{\mu_3\mu_4} - \\ & -\frac{1}{2}F_{\mu_1\mu_2}F_{\mu_2\mu_3}F_{\mu_4\mu_5}F_{\mu_3\mu_1}F_{\mu_5\mu_4} - \frac{1}{2}\,(D_{\mu_1}F_{\mu_2\mu_3})(D_{\mu_1}F_{\mu_3\mu_4})F_{\mu_5\mu_2}F_{\mu_4\mu_5} - \\ & -\frac{1}{2}(D_{\mu_1}F_{\mu_2\mu_3})F_{\mu_5\mu_2}\,(D_{\mu_1}F_{\mu_3\mu_4})F_{\mu_4\mu_5} + \\ & +\frac{1}{8}\,(D_{\mu_1}F_{\mu_2\mu_3})F_{\mu_4\mu_5}\,(D_{\mu_1}F_{\mu_3\mu_2})F_{\mu_5\mu_4} - \end{aligned}$$

$$
\begin{aligned}
&- \left(D_{\mu_5} F_{\mu_1\mu_2}\right) F_{\mu_3\mu_4} \left(D_{\mu_1} F_{\mu_2\mu_3}\right) F_{\mu_4\mu_5} + \\
&+ F_{\mu_1\mu_2} \left(D_{\mu_1} F_{\mu_3\mu_4}\right) \left(D_{\mu_5} F_{\mu_2\mu_3}\right) F_{\mu_4\mu_5}\Big) \,, \qquad (6.1)
\end{aligned}
$$

with $\lambda \in \mathbb{R}$. We omitted terms in eq. (6.1) which can be removed through field redefinitions. The terms appearing in eq. (6.1) are inert under field redefinitions.

Let us now compare our result to the existing literature. Perhaps the cleanest calculation can be found in [13]. There, a detailed analysis of the four-point open superstring amplitude was performed. This was matched to the two-derivative terms at order α'^3 in the effective action. The result of [13] for these terms, in our conventions, reads,

$$
\begin{aligned}
\mathcal{L}^{der}_{(3)} = \frac{\zeta(3)}{(2\pi)^3} \mathrm{tr}\, \Big(&[F_{\mu\nu}, D_\lambda F_{\sigma\mu}][D_\lambda F_{\nu\rho}, F_{\rho\sigma}] + [F_{\mu\nu}, D_\lambda F_{\sigma\rho}][D_\lambda F_{\nu\rho}, F_{\mu\sigma}] - \\
&- \frac{1}{2}[F_{\mu\nu}, D_\lambda F_{\rho\sigma}][D_\lambda F_{\mu\nu}, F_{\rho\sigma}]\Big) \,. \qquad (6.2)
\end{aligned}
$$

Passing to the basis for the independent terms we chose at this order, one finds that eq. (6.2), modulo FR changeable terms, exactly reproduces the derivative terms in eq. (6.1) with,

$$
\lambda = -\frac{2\zeta(3)}{\pi^3} \,, \qquad (6.3)
$$

thereby fixing our free parameter. In addition, the change of basis yields terms without derivatives as well. Indeed, we get $l^5_{0,0,0} = 0$, $l^5_{0,0,1} = -3\lambda/4$, $l^5_{0,0,2} = -\lambda/2$, $l^5_{0,0,3} = -3\lambda/4$, $l^5_{0,1,0} = \lambda/8$ and $l^5_{0,1,1} = 3\lambda/8$. This does not agree with our result in eq. (6.1). This is not surprising as, in order to fully determine these terms, the calculation of [13] has to be supplemented with the calculation and analysis of a five-point open superstring amplitude.

Subsequently we turn to the calculation of the one-loop effective action of $N = 4$ super Yang-Mills in 4 dimensions through operators of dimension 10 in [19]. As a full result at $\mathcal{O}(\alpha'^3)$ is claimed in [19], we present a detailed comparison. A particular property of $d = 4$ is that only 4 of the 6 terms without derivatives are independent. This implies that the following transformation is always possible in $d = 4$,

$$
\begin{aligned}
&l^5_{0,0,0} \to l^5_{0,0,0} + \frac{3}{5} l^5_{0,1,0} - \frac{1}{5} l^5_{0,1,1} \,, \\
&l^5_{0,0,1} \to l^5_{0,0,1} - l^5_{0,1,0} + l^5_{0,1,1} \,, \\
&l^5_{0,0,2} \to l^5_{0,0,2} + l^5_{0,1,0} + l^5_{0,1,1} \,, \\
&l^5_{0,0,3} \to l^5_{0,0,3} + \frac{1}{5} l^5_{0,1,0} + \frac{3}{5} l^5_{0,1,1} \,, \\
&l^5_{0,1,0} \to 0 \,, \\
&l^5_{0,1,1} \to 0 \,. \qquad (6.4)
\end{aligned}
$$

Restricting eq. (6.1) to four dimensions and implementing eq. (6.4) into it, we get that the terms without derivatives are changed to

$$l^5_{0,0,0} = -\frac{\lambda}{10}\,, \quad l^5_{0,0,1} = -\frac{\lambda}{2}\,, \quad l^5_{0,0,2} = \frac{\lambda}{2}\,, \quad l^5_{0,0,3} = -\frac{7\lambda}{10}\,, \quad l^5_{0,1,0} = 0,\ l^5_{0,1,1} = 0\,. \tag{6.5}$$

We now turn to eq. (6.8) in [19]. It contains four terms without derivatives whose coefficients we call, in an obvious notation, $\hat{l}^5_{(0,0,s)}$, $s \in \{0,1,2,3\}$. The terms with derivatives read in our conventions,

$$\begin{aligned}\mathcal{L}^{der}_{(3)} = \operatorname{tr}\Big(& l^5_{1,0,3} D_{\mu_1} F_{\mu_2\mu_3} D_{\mu_1} F_{\mu_3\mu_4} F_{\mu_4\mu_5} F_{\mu_5\mu_2} + l^5_{1,0,8} D_{\mu_1} F_{\mu_2\mu_3} F_{\mu_3\mu_4} F_{\mu_5\mu_2} D_{\mu_1} F_{\mu_4\mu_5} + \\ & + l^5_{1,0,4} D_{\mu_1} F_{\mu_2\mu_3} D_{\mu_1} F_{\mu_3\mu_4} F_{\mu_5\mu_2} F_{\mu_4\mu_5} + l^5_{1,1,3} D_{\mu_1} F_{\mu_2\mu_3} D_{\mu_1} F_{\mu_3\mu_2} F_{\mu_4\mu_5} F_{\mu_5\mu_4} + \\ & + l^5_{1,1,6} D_{\mu_1} F_{\mu_2\mu_3} F_{\mu_3\mu_2} F_{\mu_4\mu_5} D_{\mu_1} F_{\mu_5\mu_4} + l^5_{1,1,7} D_{\mu_1} F_{\mu_2\mu_3} D_{\mu_1} F_{\mu_4\mu_5} F_{\mu_3\mu_2} F_{\mu_5\mu_4}\Big)\,,\end{aligned} \tag{6.6}$$

where, modulo an overall multiplicative constant, [19] gives,

$$l^5_{1,0,3} = l^5_{1,0,4} = l^5_{1,0,8} = -\frac{\lambda}{4}\,, \qquad l^5_{1,1,3} = l^5_{1,1,6} = l^5_{1,1,7} = \frac{\lambda}{16}\,. \tag{6.7}$$

Before comparing, we need to rewrite the $(1,0,3)$, $(1,0,8)$, $(1,1,6)$ and $(1,1,7)$ terms in our basis. Ignoring the FR removable terms, we get the following conversion table,

$$\begin{aligned} & l^5_{0,0,0} \to l^5_{0,0,0} + l^5_{1,0,3}\,, \quad l^5_{0,0,1} \to l^5_{0,0,1} + l^5_{1,0,3} + 2l^5_{1,0,8}\,, \\ & l^5_{0,0,2} \to l^5_{0,0,2} + l^5_{1,0,3} + 2l^5_{1,0,8}\,, \quad l^5_{0,0,3} \to l^5_{0,0,3} + 3l^5_{1,0,3}\,, \\ & l^5_{0,1,0} \to l^5_{0,1,0} - \frac{1}{2} l^5_{1,0,3} + 4l^5_{1,1,6}\,, \quad l^5_{0,1,1} \to l^5_{0,1,1} - l^5_{1,0,3} + 2l^5_{1,1,7}\,, \\ & l^5_{1,0,4} \to l^5_{1,0,4} - 2l^5_{1,0,3} - l^5_{1,0,8}\,, \quad l^5_{1,0,6} \to l^5_{1,0,6} - l^5_{1,0,3} - l^5_{1,0,8}\,, \\ & l^5_{1,1,3} \to l^5_{1,1,3} - l^5_{1,1,6}\,, \quad l^5_{1,1,4} \to l^5_{1,1,4} + \frac{3}{8} l^5_{1,0,3} - \frac{1}{2} l^5_{1,1,7}\,, \\ & l^5_{1,4,30} \to l^5_{1,4,30} - 4l^5_{1,0,3}\,, \quad l^5_{1,4,47} \to l^5_{1,4,47} - l^5_{1,0,3} - 4l^5_{1,1,6}\,, \\ & l^5_{1,4,58} \to l^5_{1,4,58} + 4l^5_{1,0,3}\,. \end{aligned} \tag{6.8}$$

Upon implementing this in eq. (6.6) and the subsequent elimination of the $(0,1,0)$ and $(0,1,1)$ terms using eq. (6.4), we obtain an action of the form eq. (6.1) where the derivative terms coincide exactly! For the terms without derivatives, we get

$$\begin{aligned} & l^5_{0,0,0} = \hat{l}^5_{0,0,0} - \frac{\lambda}{10}\,, \quad l^5_{0,0,1} = \hat{l}^5_{0,0,1} - \frac{3\lambda}{4}\,, \quad l^5_{0,0,2} = \hat{l}^5_{0,0,2}\,, \\ & l^5_{0,0,3} = \hat{l}^5_{0,0,3} - \frac{9\lambda}{20}\,, \quad l^5_{0,1,0} = l^5_{0,1,1} = 0\,. \end{aligned} \tag{6.9}$$

Matching this to our result eq. (6.5), we find that we need,

$$\hat{l}^5_{0,0,0} = 0\,, \qquad \hat{l}^5_{0,0,1} = \frac{\lambda}{4}\,, \qquad \hat{l}^5_{0,0,2} = \frac{\lambda}{2}\,, \qquad \hat{l}^5_{0,0,3} = -\frac{5\lambda}{20}\,. \tag{6.10}$$

One easily checks that this does *not* agree with [19]!

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Finally, there is the direct calculation in [12]. We verified that the terms with derivatives do again coincide with our derivative terms. However the precise structure of the terms without derivatives in [12] remains obscure. We compared various readings of [12] and always found disagreement with our result as well as with the result in [19].

A consequence of all this is that we can be very confident about the derivative terms in eq. (6.1). It agrees perfectly with direct calculations, [13], [12] and the $d = 4$ $N = 4$ super Yang-Mills effective action calculation [19]. In addition, the precise comparison of eq. (6.1) to [13] fixes the free parameter λ as in eq. (6.3). Concerning the terms without derivatives, no agreement exists between our calculation and the result in [19]. This shows that in general one should not expect a direct relation between the tree-level open string effective action, which is calculated here, and the quantum super Yang-Mills effective action (for a more detailed discussion, we refer to [26]), which is studied in [19]. In fact, already in the abelian case, it is known that the F^8 term in the one-loop $N = 4$ super Yang-Mills effective action is different in structure, [26], from the F^8 term in the Born-Infeld action [27]. We are confident that eq. (6.1), modulo field redefinitions, together with eq. (6.3) is the non-abelian Born-Infeld action at $\mathcal{O}(\alpha'^3)$. Indeed, the only ansatz we made is that configurations satisfying eq. (2.4) and some deformation of eq. (2.5) solve the equations of motion. In both the abelian limit and in the limit that $2\pi\alpha' F$ is small, such solutions are known to represent BPS configurations of D-branes. It is hard to conceive that such configurations would cease to exist away from these limits. In other words, if these BPS configurations exist in the non-abelian case where $2\pi\alpha' F$ is not necessarily small, then $\mathcal{L} = \mathcal{L}_{(0)} + \mathcal{L}_{(2)} + \mathcal{L}_{(3)}$, with $\mathcal{L}_{(0)}$, $\mathcal{L}_{(2)}$ and $\mathcal{L}_{(3)}$ given in eqs. (2.2), (3.13) and (6.1) resp., should be the non-abelian Born-Infeld action through order α'^3!

A very interesting and strong check of our result would follow the program set up in [10] and further developed in [14]. There the mass spectrum of D-branes at angles is calculated. Upon T-dualizing, this corresponds to stacks of D-branes in the presence of constant magnetic background fields. The non-abelian Born-Infeld action should reproduce this spectrum. In particular, the string theoretic calculation shows that the spectrum of the off-diagonal gauge fields receives only contributions at even powers in α'. This means that the contribution to the spectrum of the terms without derivatives in eq. (6.1) should exactly cancel against the contributions coming from the terms with derivatives! Work in this direction is in progress.[10]

The result in eq. (6.1) is not sufficient to make all order predictions about the structure of the non-abelian Born-Infeld action. Presently, our software has been optimized to tackle the Born-Infeld action at the next order. Hopefully this will shed some light on the all-order structure of the non-abelian Born-Infeld action.

[10]Note added in proof: In meanwhile this program has been carried out in [28]. The result in eq. (6.1) indeed correctly reproduces the spectrum, while the results of [12] and [19] do not.

The fact that we obtain a one-parameter family of allowed deformations at order α'^3 suggests that supersymmetry alone is not sufficient to fix the full non-abelian Born-Infeld action. In this context it would be most interesting to push the analysis of [17] one order higher. Note that it is very fortunate that we did obtain a one parameter family of solutions. Indeed, it is clear that, in units where $2\pi\alpha' = 1$, our method can only fix coefficients in terms of rational numbers. From open superstring amplitudes, one expects that the coefficients at order α'^n are given by $\zeta(n)/\pi^n$ times a rational number. For n even this is a rational number while for n odd it is not! So it would not be too surprising that our method would fix the deformation completely at even orders in α', while giving a one-parameter family of deformations at odd orders.

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A. Notations and conventions

Our metric is euclidean. Indices denoted by μ, ν, ... run from 1 to $2p$ and those denoted by α, β, ... run from 1 to p. We choose anti-hermitian matrices for the $u(n)$ generators. The fieldstrength and covariant derivative are given by

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]\,, \\ D_\mu \cdot &= \partial_\mu \cdot + [A_\mu, \cdot]\,. \end{aligned} \tag{A.1}$$

Instead of using real spatial coordinates x^μ, $\mu \in \{1, \ldots, 2p\}$, we will often use complex coordinates z^α, $\alpha \in \{1, \ldots p\}$,

$$z^\alpha \equiv \frac{1}{\sqrt{2}}\left(x^{2\alpha-1} + i x^{2\alpha}\right), \qquad \bar{z}^{\bar{\alpha}} \equiv \frac{1}{\sqrt{2}}\left(x^{2\alpha-1} - i x^{2\alpha}\right). \tag{A.2}$$

As we work in flat space, the metric is $g_{\alpha\beta} = g_{\bar{\alpha}\bar{\beta}} = 0$, $g_{\alpha\bar{\beta}} = \delta_{\alpha\bar{\beta}}$.

Finally, we explain our strategy in classifying terms in the action, the equations of motion, ... Each term contains a number of derivatives $n(D)$ and fieldstrengths $n(F)$. Terms are classified according to (hierarchically):

1. *order* n: for terms in the lagrangian the order can be calculated by:

$$n = n(F) + \frac{n(D)}{2} \,. \tag{A.3}$$

   This corresponds in fact to order $\alpha'^{(n-2)}$. For terms in the equations of motion, we define the order as the order of the terms in the lagrangian from which the terms are derived. We can still use formula (A.3) if we count the free index as an extra derivative.

2. *superstructure* s_1: within a certain order, one can classify the terms according to the number of derivatives. In the abelian case, the different superstructures do not communicate; in the non-abelian case there are identities connecting them. They read in their most general form:

$$D_1 \dots D_k[D_{k+1}, D_{k+2}]D_{k+3} \dots D_n F_{l_1 l_2} = D_1 \dots D_k[F_{k+1;k+2}, D_{k+3} \dots D_n F_{l_1 l_2}] \,. \tag{A.4}$$

   In the rest of the article we will use the shorthand notation $[D, D]\cdot = [F, \cdot]$.

3. *index structure* s_2: it is convenient to use an example to explain this. At α'^2, we have the following four terms without derivatives:

$$l^4_{0,0,0}\ F_{\mu_1\mu_a2} F_{\mu_2\mu_3} F_{\mu_3\mu_4} F_{\mu_4\mu_1} \,,\ l^4_{0,0,1}\ F_{\mu_1\mu_2} F_{\mu_2\mu_3} F_{\mu_4\mu_1} F_{\mu_3\mu_4} \,, \tag{A.5a}$$
$$l^4_{0,1,0}\ F_{\mu_1\mu_2} F_{\mu_2\mu_1} F_{\mu_3\mu_4} F_{\mu_4\mu_3} \,,\ l^4_{0,1,1}\ F_{\mu_1\mu_2} F_{\mu_3\mu_4} F_{\mu_2\mu_1} F_{\mu_4\mu_3} . \tag{A.5b}$$

   We note that the first two contain a loop[11] with 4 F's, while the latter two contain two loops with 2 F's each. For the first two we take $s_2 = 0$, while the latter we label by $s_2 = 1$. So s_2 will distinguish terms with different loop structure. When classifying terms with derivatives e.g. in the equations of motion, one will also have "chains". Those are Lorentz contracted sets of fieldstrengths with two indices contracted with derivatives or one index contracted with a derivative and the other one a free index. We will call the latter type "a free index chain". Two examples containing chains, one from the lagrangian and one from the equations of motion are:

$$l^4_{1,2,1}\ \left(D_{\mu_4} D_{\mu_1} F_{\mu_1\mu_2}\right) F_{\mu_3\mu_4} F_{\mu_2\mu_3} \tag{A.5c}$$
$$e^4_{0,0,0}\ \left(D_{\bar\beta} F_{\alpha_1\bar\alpha_2}\right) F_{\alpha_2\bar\alpha_3} F_{\alpha_3\bar\alpha_1} \tag{A.5d}$$

   Note that the index structure of a term can be elegantly represented by a graph: see figure 1.

[11] An n-loop contains n fieldstrengths, traced over the Lorentz indices and disregarding any ordering.

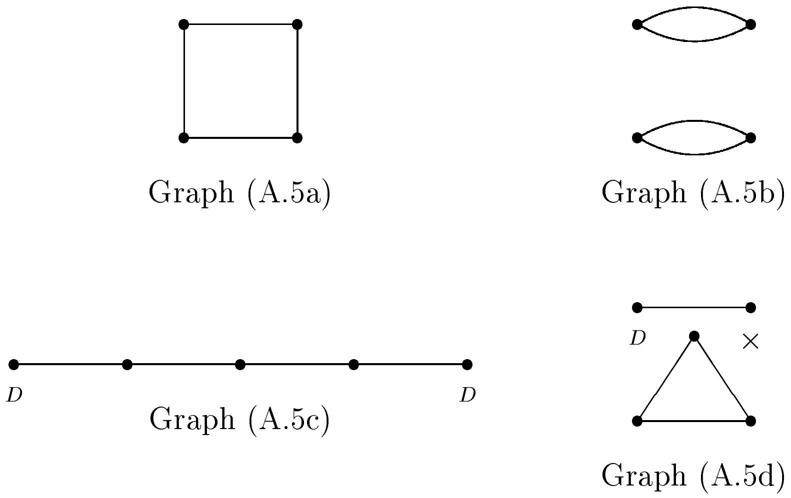

Figure 1: Graphs for the terms of (A.5a)-(A.5d). The free index is denoted by a $\times$. Note that in term (A.5d) the free index chain contains zero F's.

4. *term number* s_3. Other classifications could be thought of, like the derivative structure, which denote on which terms the derivatives act. In the non-abelian case the order of the F's is also important. We take these two together and just number the different terms within an index structure. For its particular value, we take that which is used in the program. As the program starts by writing down all possible terms and subsequently eliminates the dependent ones through partial integration, Bianchi identities, etc. the concrete values of s_3 will not necessarily be in numerical order.

Concluding, a term in the lagrangian will be labelled by $l^n_{s_1,s_2,s_3}$. In a similar way we label terms in the equations of motion by $e^n_{s_1,s_2,s_3}$, those in the field redefinitions by $f^n_{s_1,s_2,s_3}$ and those in the deformations of the DUY condition, eq. (2.5), by $d^n_{s_1,s_2,s_3}$.

B. The equations of motion at order α'^2

In this appendix, we give all contributions to the equations of motion at order α'^2. They follow in a straightforward way from eq. (3.6). In addition to this, we take the effects of deforming eq. (2.5) into account. At this order, the most general deformation of the stability condition consistent with our assumption stated in section 2, reads as

$$F_{\alpha\bar{\alpha}} + \mathcal{D}_{(2)} + \mathcal{O}(\alpha'^3) = 0\,, \tag{B.1}$$

with

$$\begin{aligned}\mathcal{D}_{(2)} = {} & d^4_{0,0,0} F_{\alpha_1\bar{\alpha}_2} F_{\alpha_2\bar{\alpha}_3} F_{\alpha_3\bar{\alpha}_1} + d^4_{0,0,1} F_{\alpha_1\bar{\alpha}_2} F_{\alpha_3\bar{\alpha}_1} F_{\alpha_2\bar{\alpha}_3} + \\ & + d^4_{1,0,4} \left(D_{\alpha_1} F_{\alpha_2\bar{\alpha}_3}\right)\left(D_{\bar{\alpha}_1} F_{\alpha_3\bar{\alpha}_2}\right) + d^4_{1,0,5} \left(D_{\bar{\alpha}_1} F_{\alpha_2\bar{\alpha}_3}\right)\left(D_{\alpha_1} F_{\alpha_3\bar{\alpha}_2}\right).\end{aligned} \tag{B.2}$$

The leading term in the equations of motion, $D_{\bar\beta}F_{\alpha\bar\alpha} = 0$, vanished because of eq. (2.5). If we now deform eq. (2.5) as in eq. (B.2), we will induce further contributions to the equations of motion.

Below, we list the contribution to the equations of motion following from $\mathcal{L}_{(2)}$, eq. (3.6) and the corrections following from $\mathcal{D}_{(2)}$, eq. (B.2). We list them below in hierarchical order (as was explained in appendix A) and omit terms with more than one 1-loop (see table 2).

Superstructure 0: #derivatives: 1, #Fs: 3

Structure 0: Free index chain length: 0 Chains: 0 Loops: 3

$$\begin{aligned}
e^4_{0,0,0} :\ & \left(D_{\bar\beta}F_{\alpha_1\bar\alpha_2}\right) F_{\alpha_2\bar\alpha_3}F_{\alpha_3\bar\alpha_1} + \\
& +2\ l^4_{0,0,1} -\ d^4_{0,0,0} - 2\ l^4_{1,2,1} \\
e^4_{0,0,1} :\ & \left(D_{\bar\beta}F_{\alpha_1\bar\alpha_2}\right) F_{\alpha_3\bar\alpha_1}F_{\alpha_2\bar\alpha_3} + \\
& +4\ l^4_{0,0,0} -\ d^4_{0,0,1} + 2\ l^4_{1,2,1} \\
e^4_{0,0,2} :\ & F_{\alpha_1\bar\alpha_2}\left(D_{\bar\beta}F_{\alpha_3\bar\alpha_1}\right) F_{\alpha_2\bar\alpha_3} - \\
& -2\ l^4_{0,0,1} -\ d^4_{0,0,1} - 16\ l^4_{0,1,1} - 8\ l^4_{0,1,0} \\
e^4_{0,0,3} :\ & F_{\alpha_1\bar\alpha_2}\left(D_{\bar\beta}F_{\alpha_2\bar\alpha_3}\right) F_{\alpha_3\bar\alpha_1} + \\
& +6\ l^4_{0,0,1} -\ d^4_{0,0,0} + 16\ l^4_{0,1,1} + 8\ l^4_{0,1,0} \\
e^4_{0,0,4} :\ & F_{\alpha_1\bar\alpha_2}F_{\alpha_2\bar\alpha_3}\left(D_{\bar\beta}F_{\alpha_3\bar\alpha_1}\right) + \\
& +4\ l^4_{0,0,0} -\ d^4_{0,0,0} + 2\ l^4_{1,2,1} + 4\ l^4_{0,0,1} + 16\ l^4_{0,1,1} + 8\ l^4_{0,1,0} \\
e^4_{0,0,5} :\ & F_{\alpha_1\bar\alpha_2}F_{\alpha_3\bar\alpha_1}\left(D_{\bar\beta}F_{\alpha_2\bar\alpha_3}\right) - \\
& -2\ l^4_{0,0,1} -\ d^4_{0,0,1} - 2\ l^4_{1,2,1} - 16\ l^4_{0,1,1} - 8\ l^4_{0,1,0}
\end{aligned} \tag{B.3}$$

Structure 1: Free index chain length: 0 Chains: 0 Loops: 2 1

$$\begin{aligned}
e^4_{0,1,0} :\ & \left(D_{\bar\beta}F_{\alpha_1\bar\alpha_2}\right) F_{\alpha_2\bar\alpha_1}F_{\alpha_3\bar\alpha_3} + \\
& +\ l^4_{1,2,1} -\ l^4_{1,2,6} - 4\ l^4_{2,1,23} \\
e^4_{0,1,1} :\ & \left(D_{\bar\beta}F_{\alpha_1\bar\alpha_2}\right) F_{\alpha_3\bar\alpha_3}F_{\alpha_2\bar\alpha_1} - \\
& -2\ l^4_{0,0,1} -\ l^4_{1,2,1} \\
e^4_{0,1,2} :\ & F_{\alpha_1\bar\alpha_2}\left(D_{\bar\beta}F_{\alpha_2\bar\alpha_1}\right) F_{\alpha_3\bar\alpha_3} + \\
& +\ l^4_{1,2,1} +\ l^4_{1,2,6} + 4\ l^4_{2,1,23} + 4\ l^4_{0,0,1} + 16\ l^4_{0,1,1} + 8\ l^4_{0,1,0} \\
e^4_{0,1,3} :\ & F_{\alpha_1\bar\alpha_1}\left(D_{\bar\beta}F_{\alpha_2\bar\alpha_3}\right) F_{\alpha_3\bar\alpha_2} + \\
& +2\ l^4_{0,0,1} +\ l^4_{1,2,6} + 4\ l^4_{2,1,23} \\
e^4_{0,1,4} :\ & F_{\alpha_1\bar\alpha_2}F_{\alpha_3\bar\alpha_3}\left(D_{\bar\beta}F_{\alpha_2\bar\alpha_1}\right) - \\
& -2\ l^4_{1,2,1} - 4\ l^4_{0,0,1} - 16\ l^4_{0,1,1} - 8\ l^4_{0,1,0} \\
e^4_{0,1,5} :\ & F_{\alpha_1\bar\alpha_1}F_{\alpha_2\bar\alpha_3}\left(D_{\bar\beta}F_{\alpha_3\bar\alpha_2}\right) + \\
& +\ l^4_{1,2,1} -\ l^4_{1,2,6} - 4\ l^4_{2,1,23}
\end{aligned}$$

$$
\begin{aligned}
e^4_{0,1,6} :\ & F_{\alpha_1\bar{\alpha}_2} F_{\alpha_2\bar{\alpha}_1} \left(D_{\bar{\beta}} F_{\alpha_3\bar{\alpha}_3}\right) + \\
& +2\ l^4_{0,0,1} -\ l^4_{1,2,6} - 4\ l^4_{2,1,23} + 16\ l^4_{0,1,0} + 16\ l^4_{0,1,1} +\ l^4_{1,2,1} \\
e^4_{0,1,7} :\ & F_{\alpha_1\bar{\alpha}_2} \left(D_{\bar{\beta}} F_{\alpha_3\bar{\alpha}_3}\right) F_{\alpha_2\bar{\alpha}_1} - \\
& -2\ l^4_{1,2,1} - 4\ l^4_{0,0,1} - 8\ l^4_{0,1,0} \\
e^4_{0,1,8} :\ & \left(D_{\bar{\beta}} F_{\alpha_1\bar{\alpha}_1}\right) F_{\alpha_2\bar{\alpha}_3} F_{\alpha_3\bar{\alpha}_2} + \\
& +\ l^4_{1,2,1} +\ l^4_{1,2,6} + 4\ l^4_{2,1,23} + 8\ l^4_{0,1,0} + 2\ l^4_{0,0,1}
\end{aligned} \tag{B.4}
$$

Structure 2: Free index chain length: 0 Chains: 0 Loops: 1 1 1
Terms omitted.
Structure 3: Free index chain length: 1 Chains: 1 Loops: 2

$$
\begin{aligned}
e^4_{0,3,0} :\ & F_{\alpha_1\bar{\beta}} \left(D_{\bar{\alpha}_1} F_{\alpha_2\bar{\alpha}_3}\right) F_{\alpha_3\bar{\alpha}_2} + \\
& +4\ l^4_{0,0,0} + 8\ l^4_{0,1,0} \\
e^4_{0,3,2} :\ & \left(D_{\bar{\alpha}_1} F_{\alpha_2\bar{\alpha}_3}\right) F_{\alpha_1\bar{\beta}} F_{\alpha_3\bar{\alpha}_2} + \\
& +4\ l^4_{0,0,1} + 16\ l^4_{0,1,1} + 8\ l^4_{0,1,0} \\
e^4_{0,3,5} :\ & F_{\alpha_2\bar{\alpha}_3} \left(D_{\bar{\alpha}_1} F_{\alpha_3\bar{\alpha}_2}\right) F_{\alpha_1\bar{\beta}} + \\
& +4\ l^4_{0,0,0} + 16\ l^4_{0,1,0} + 4\ l^4_{0,0,1} + 16\ l^4_{0,1,1}
\end{aligned} \tag{B.5}
$$

Structure 5: Free index chain length: 2 Chains: 2 Loops: 1

$$
\begin{aligned}
e^4_{0,5,2} :\ & F_{\alpha_2\bar{\beta}} F_{\alpha_1\bar{\alpha}_2} \left(D_{\bar{\alpha}_1} F_{\alpha_3\bar{\alpha}_3}\right) + \\
& +4\ l^4_{0,0,1} + 4\ l^4_{0,0,0} \\
e^4_{0,5,3} :\ & \left(D_{\bar{\alpha}_1} F_{\alpha_3\bar{\alpha}_3}\right) F_{\alpha_1\bar{\alpha}_2} F_{\alpha_2\bar{\beta}} + \\
& +4\ l^4_{0,0,1} + 4\ l^4_{0,0,0}
\end{aligned} \tag{B.6}
$$

Superstructure 1: #derivatives: 3, #Fs: 2

Structure 0: Free index chain length: 0 Chains: 0 0 Loops: 2

$$
\begin{aligned}
e^4_{1,0,7} :\ & F_{\alpha_2\bar{\alpha}_3} \left(D_{\alpha_1} D_{\bar{\beta}} D_{\bar{\alpha}_1} F_{\alpha_3\bar{\alpha}_2}\right) + \\
& +4\ l^4_{0,0,1} + 16\ l^4_{0,1,1} + 8\ l^4_{0,1,0} \\
e^4_{1,0,16} :\ & \left(D_{\bar{\beta}} D_{\alpha_1} F_{\alpha_2\bar{\alpha}_3}\right) \left(D_{\bar{\alpha}_1} F_{\alpha_3\bar{\alpha}_2}\right) - \\
& -\ d^4_{1,0,4} - 2\ l^4_{0,0,1} - 8\ l^4_{0,1,0} -\ l^4_{1,2,1} \\
e^4_{1,0,17} :\ & \left(D_{\bar{\alpha}_1} F_{\alpha_2\bar{\alpha}_3}\right) \left(D_{\bar{\beta}} D_{\alpha_1} F_{\alpha_3\bar{\alpha}_2}\right) + \\
& +\ l^4_{1,2,1} -\ d^4_{1,0,5} + 2\ l^4_{0,0,1} + 8\ l^4_{0,1,0} \\
e^4_{1,0,18} :\ & \left(D_{\bar{\beta}} D_{\bar{\alpha}_1} F_{\alpha_2\bar{\alpha}_3}\right) \left(D_{\alpha_1} F_{\alpha_3\bar{\alpha}_2}\right) - \\
& -\ d^4_{1,0,5} +\ l^4_{1,2,1} \\
e^4_{1,0,19} :\ & \left(D_{\alpha_1} F_{\alpha_2\bar{\alpha}_3}\right) \left(D_{\bar{\beta}} D_{\bar{\alpha}_1} F_{\alpha_3\bar{\alpha}_2}\right) - \\
& -\ l^4_{1,2,1} -\ d^4_{1,0,4}
\end{aligned}
$$

$$
\begin{aligned}
e^4_{1,0,20} &: \left(D_{\alpha_1} D_{\bar\beta} F_{\alpha_2\bar\alpha_3}\right)\left(D_{\bar\alpha_1} F_{\alpha_3\bar\alpha_2}\right) + \\
&\quad +2\ l^4_{1,2,1} + 2\ l^4_{0,0,1} + 8\ l^4_{0,1,0} \\
e^4_{1,0,21} &: \left(D_{\bar\alpha_1} F_{\alpha_2\bar\alpha_3}\right)\left(D_{\alpha_1} D_{\bar\beta} F_{\alpha_3\bar\alpha_2}\right) - \\
&\quad -2\ l^4_{1,2,1} - 2\ l^4_{0,0,1} - 8\ l^4_{0,1,0}
\end{aligned}
\tag{B.7}
$$

Structure 1: Free index chain length: 0 Chains: 0 0 Loops: 1 1
Terms omitted.
Structure 3: Free index chain length: 0 Chains: 1 0 Loops: 1

$$
\begin{aligned}
e^4_{1,3,4} &: \left(D_{\alpha_1} D_{\bar\beta} F_{\alpha_2\bar\alpha_1}\right)\left(D_{\bar\alpha_2} F_{\alpha_3\bar\alpha_3}\right) - \\
&\quad -2\ l^4_{1,2,6} - 8\ l^4_{2,1,23} - 4\ l^4_{0,0,1} -\ l^4_{1,2,1} \\
e^4_{1,3,5} &: \left(D_{\bar\alpha_2} F_{\alpha_3\bar\alpha_3}\right)\left(D_{\alpha_1} D_{\bar\beta} F_{\alpha_2\bar\alpha_1}\right) + \\
&\quad +2\ l^4_{1,2,6} + 8\ l^4_{2,1,23} + 4\ l^4_{0,0,1} +\ l^4_{1,2,1} \\
e^4_{1,3,8} &: \left(D_{\bar\beta} F_{\alpha_2\bar\alpha_1}\right)\left(D_{\bar\alpha_2} D_{\alpha_1} F_{\alpha_3\bar\alpha_3}\right) - \\
&\quad -8\ l^4_{2,1,23} - 2\ l^4_{1,2,6} + 2\ l^4_{1,2,1} \\
e^4_{1,3,9} &: \left(D_{\bar\alpha_2} D_{\alpha_1} F_{\alpha_3\bar\alpha_3}\right)\left(D_{\bar\beta} F_{\alpha_2\bar\alpha_1}\right) + \\
&\quad +8\ l^4_{2,1,23} + 2\ l^4_{1,2,6} - 2\ l^4_{1,2,1} \\
e^4_{1,3,13} &: \left(D_{\bar\beta} D_{\bar\alpha_2} D_{\alpha_1} F_{\alpha_3\bar\alpha_3}\right) F_{\alpha_2\bar\alpha_1} + \\
&\quad +\ l^4_{1,2,1} - 2\ l^4_{0,0,1} \\
e^4_{1,3,14} &: F_{\alpha_1\bar\alpha_2}\left(D_{\bar\beta} D_{\alpha_2} D_{\bar\alpha_1} F_{\alpha_3\bar\alpha_3}\right) - \\
&\quad -2\ l^4_{0,0,1} -\ l^4_{1,2,1} - 16\ l^4_{0,1,1} - 8\ l^4_{0,1,0} \\
e^4_{1,3,20} &: F_{\alpha_2\bar\alpha_1}\left(D_{\bar\alpha_2} D_{\alpha_1} D_{\bar\beta} F_{\alpha_3\bar\alpha_3}\right) - \\
&\quad -2\ l^4_{1,2,6} - 8\ l^4_{2,1,23} - 2\ l^4_{0,0,1} -\ l^4_{1,2,1} \\
e^4_{1,3,23} &: \left(D_{\alpha_2} D_{\bar\alpha_1} D_{\bar\beta} F_{\alpha_3\bar\alpha_3}\right) F_{\alpha_1\bar\alpha_2} + \\
&\quad +2\ l^4_{1,2,6} + 8\ l^4_{2,1,23} + 2\ l^4_{0,0,1} +\ l^4_{1,2,1}
\end{aligned}
\tag{B.8}
$$

Superstructure 2: #derivatives: 5, #Fs: 1

Structure 0: Free index chain length: 0 Chains: 0 0 0 Loops: 1

$$
\begin{aligned}
e^4_{2,0,0} &: \left(D_{\bar\beta} D_{\bar\alpha_2} D_{\alpha_2} D_{\bar\alpha_1} D_{\alpha_1} F_{\alpha_3\bar\alpha_3}\right) + \\
&\quad +8\ l^4_{2,1,23} -\ l^4_{1,2,1} \\
e^4_{2,0,6} &: \left(D_{\alpha_2} D_{\bar\beta} D_{\bar\alpha_2} D_{\bar\alpha_1} D_{\alpha_1} F_{\alpha_3\bar\alpha_3}\right) - \\
&\quad -2\ l^4_{1,2,6} - 8\ l^4_{2,1,23} \\
e^4_{2,0,48} &: \left(D_{\bar\alpha_2} D_{\alpha_2} D_{\bar\alpha_1} D_{\alpha_1} D_{\bar\beta} F_{\alpha_3\bar\alpha_3}\right) + \\
&\quad +\ l^4_{1,2,1} + 2\ l^4_{1,2,6} + 8\ l^4_{2,1,23}
\end{aligned}
\tag{B.9}
$$

C. Lagrangian and DUY deformation at order α'^3

In this appendix we will quote a base for the most general lagrangian and DUY deformation at order α'^3 for further use in the text. These were calculated by our computer program following the method described in section 4.

Below we first list the lagrangian $\mathcal{L}_{(3)}$, with terms sensitive to field redefinitions marked by (FR). For the numbering logic, see appendix A.

Superstructure 0: #derivatives: 0, #Fs: 5

Structure 0: Loops: 5

$$\begin{aligned}
l^5_{0,0,0} &: F_{\mu_1\mu_2}F_{\mu_2\mu_3}F_{\mu_3\mu_4}F_{\mu_4\mu_5}F_{\mu_5\mu_1} \\
l^5_{0,0,1} &: F_{\mu_1\mu_2}F_{\mu_2\mu_3}F_{\mu_3\mu_4}F_{\mu_5\mu_1}F_{\mu_4\mu_5} \\
l^5_{0,0,2} &: F_{\mu_1\mu_2}F_{\mu_2\mu_3}F_{\mu_5\mu_1}F_{\mu_3\mu_4}F_{\mu_4\mu_5} \\
l^5_{0,0,3} &: F_{\mu_1\mu_2}F_{\mu_4\mu_5}F_{\mu_2\mu_3}F_{\mu_5\mu_1}F_{\mu_3\mu_4}
\end{aligned} \tag{C.1}$$

Structure 1: Loops: 3 2

$$\begin{aligned}
l^5_{0,1,0} &: F_{\mu_1\mu_2}F_{\mu_2\mu_3}F_{\mu_3\mu_1}F_{\mu_4\mu_5}F_{\mu_5\mu_4} \\
l^5_{0,1,1} &: F_{\mu_1\mu_2}F_{\mu_2\mu_3}F_{\mu_4\mu_5}F_{\mu_3\mu_1}F_{\mu_5\mu_4}
\end{aligned} \tag{C.2}$$

Superstructure 1: #derivatives: 2, #Fs: 4

Structure 0: Chains: 0 Loops: 4

$$\begin{aligned}
l^5_{1,0,4} &: (D_{\mu_1}F_{\mu_2\mu_3})\,(D_{\mu_1}F_{\mu_3\mu_4})\,F_{\mu_5\mu_2}F_{\mu_4\mu_5} \\
l^5_{1,0,6} &: (D_{\mu_1}F_{\mu_2\mu_3})\,F_{\mu_5\mu_2}\,(D_{\mu_1}F_{\mu_3\mu_4})\,F_{\mu_4\mu_5}
\end{aligned} \tag{C.3}$$

Structure 1: Chains: 0 Loops: 2 2

$$\begin{aligned}
l^5_{1,1,3} &: (D_{\mu_1}F_{\mu_2\mu_3})\,(D_{\mu_1}F_{\mu_3\mu_2})\,F_{\mu_4\mu_5}F_{\mu_5\mu_4} \\
l^5_{1,1,4} &: (D_{\mu_1}F_{\mu_2\mu_3})\,F_{\mu_4\mu_5}\,(D_{\mu_1}F_{\mu_3\mu_2})\,F_{\mu_5\mu_4}
\end{aligned} \tag{C.4}$$

Structure 3: Chains: 2 Loops: 2

$$\begin{aligned}
l^5_{1,3,0} &: \; (D_{\mu_3}D_{\mu_1}F_{\mu_1\mu_2})\,F_{\mu_2\mu_3}F_{\mu_4\mu_5}F_{\mu_5\mu_4}\;(\mathrm{FR}) \\
l^5_{1,3,1} &: \; (D_{\mu_3}D_{\mu_1}F_{\mu_1\mu_2})\,F_{\mu_4\mu_5}F_{\mu_2\mu_3}F_{\mu_5\mu_4}\;(\mathrm{FR}) \\
l^5_{1,3,2} &: \; (D_{\mu_3}D_{\mu_1}F_{\mu_1\mu_2})\,F_{\mu_4\mu_5}F_{\mu_5\mu_4}F_{\mu_2\mu_3}\;(\mathrm{FR}) \\
l^5_{1,3,10} &: (D_{\mu_1}F_{\mu_1\mu_2})\,(D_{\mu_3}F_{\mu_4\mu_5})\,F_{\mu_2\mu_3}F_{\mu_5\mu_4}\;(\mathrm{FR})
\end{aligned} \tag{C.5}$$

Structure 4: Chains: 4

$$
\begin{aligned}
l^5_{1,4,0} &: (D_{\mu_5}D_{\mu_1}F_{\mu_1\mu_2})\,F_{\mu_2\mu_3}F_{\mu_3\mu_4}F_{\mu_4\mu_5}\ (\mathrm{FR})\\
l^5_{1,4,1} &: (D_{\mu_5}D_{\mu_1}F_{\mu_1\mu_2})\,F_{\mu_2\mu_3}F_{\mu_4\mu_5}F_{\mu_3\mu_4}\ (\mathrm{FR})\\
l^5_{1,4,2} &: (D_{\mu_5}D_{\mu_1}F_{\mu_1\mu_2})\,F_{\mu_3\mu_4}F_{\mu_2\mu_3}F_{\mu_4\mu_5}\ (\mathrm{FR})\\
l^5_{1,4,3} &: (D_{\mu_5}D_{\mu_1}F_{\mu_1\mu_2})\,F_{\mu_4\mu_5}F_{\mu_2\mu_3}F_{\mu_3\mu_4}\ (\mathrm{FR})\\
l^5_{1,4,5} &: (D_{\mu_5}D_{\mu_1}F_{\mu_1\mu_2})\,F_{\mu_4\mu_5}F_{\mu_3\mu_4}F_{\mu_2\mu_3}\ (\mathrm{FR})\\
l^5_{1,4,12} &: (D_{\mu_1}F_{\mu_1\mu_2})\,(D_{\mu_5}F_{\mu_2\mu_3})\,F_{\mu_3\mu_4}F_{\mu_4\mu_5}\ (\mathrm{FR})\\
l^5_{1,4,13} &: (D_{\mu_1}F_{\mu_1\mu_2})\,(D_{\mu_5}F_{\mu_2\mu_3})\,F_{\mu_4\mu_5}F_{\mu_3\mu_4}\ (\mathrm{FR})\\
l^5_{1,4,14} &: (D_{\mu_1}F_{\mu_1\mu_2})\,F_{\mu_3\mu_4}\,(D_{\mu_5}F_{\mu_2\mu_3})\,F_{\mu_4\mu_5}\ (\mathrm{FR})\\
l^5_{1,4,15} &: (D_{\mu_1}F_{\mu_1\mu_2})\,F_{\mu_4\mu_5}\,(D_{\mu_5}F_{\mu_2\mu_3})\,F_{\mu_3\mu_4}\ (\mathrm{FR})\\
l^5_{1,4,17} &: (D_{\mu_1}F_{\mu_1\mu_2})\,F_{\mu_4\mu_5}F_{\mu_3\mu_4}\,(D_{\mu_5}F_{\mu_2\mu_3})\ (\mathrm{FR})\\
l^5_{1,4,24} &: (D_{\mu_1}F_{\mu_1\mu_2})\,F_{\mu_2\mu_3}F_{\mu_3\mu_4}\,(D_{\mu_5}F_{\mu_4\mu_5})\ (\mathrm{FR})\\
l^5_{1,4,25} &: (D_{\mu_1}F_{\mu_1\mu_2})\,F_{\mu_2\mu_3}\,(D_{\mu_5}F_{\mu_4\mu_5})\,F_{\mu_3\mu_4}\ (\mathrm{FR})\\
l^5_{1,4,27} &: (D_{\mu_1}F_{\mu_1\mu_2})\,F_{\mu_3\mu_4}\,(D_{\mu_5}F_{\mu_4\mu_5})\,F_{\mu_2\mu_3}\ (\mathrm{FR})\\
l^5_{1,4,30} &: (D_{\mu_5}F_{\mu_1\mu_2})\,F_{\mu_3\mu_4}\,(D_{\mu_1}F_{\mu_2\mu_3})\,F_{\mu_4\mu_5}\\
l^5_{1,4,47} &: F_{\mu_1\mu_2}\,(D_{\mu_1}F_{\mu_2\mu_3})\,F_{\mu_4\mu_5}\,(D_{\mu_5}F_{\mu_3\mu_4})\\
l^5_{1,4,58} &: F_{\mu_1\mu_2}\,(D_{\mu_1}F_{\mu_3\mu_4})\,(D_{\mu_5}F_{\mu_2\mu_3})\,F_{\mu_4\mu_5}
\end{aligned} \tag{C.6}
$$

Superstructure 2: #derivatives: 4, #Fs: 3

Structure 2: Chains: 3 0

$$
\begin{aligned}
l^5_{2,2,66} &: (D_{\mu_5}D_{\mu_1}F_{\mu_1\mu_2})\,(D_{\mu_5}D_{\mu_4}F_{\mu_2\mu_3})\,F_{\mu_3\mu_4}\ (\mathrm{FR})\\
l^5_{2,2,91} &: (D_{\mu_5}D_{\mu_5}D_{\mu_1}F_{\mu_1\mu_2})\,(D_{\mu_4}F_{\mu_3\mu_4})\,F_{\mu_2\mu_3}\ (\mathrm{FR})
\end{aligned} \tag{C.7}
$$

Structure 3: Chains: 2 1

$$
\begin{aligned}
l^5_{2,3,39} &: (D_{\mu_3}D_{\mu_4}D_{\mu_1}F_{\mu_1\mu_2})\,(D_{\mu_5}F_{\mu_4\mu_5})\,F_{\mu_2\mu_3}\ (\mathrm{FR})\\
l^5_{2,3,70} &: (D_{\mu_4}D_{\mu_1}F_{\mu_1\mu_2})\,(D_{\mu_3}F_{\mu_2\mu_3})\,(D_{\mu_5}F_{\mu_4\mu_5})\ (\mathrm{FR})\\
l^5_{2,3,97} &: (D_{\mu_1}F_{\mu_1\mu_2})\,(D_{\mu_4}F_{\mu_2\mu_3})\,(D_{\mu_3}D_{\mu_5}F_{\mu_4\mu_5})\ (\mathrm{FR})
\end{aligned} \tag{C.8}
$$

Superstructure 3: #derivatives: 6, #Fs: 2

Structure 1: Chains: 2 0 0

$$
l^5_{3,1,225} : (D_{\mu_5}D_{\mu_4}D_{\mu_4}D_{\mu_1}F_{\mu_1\mu_2})\,(D_{\mu_5}D_{\mu_3}F_{\mu_2\mu_3})\ (\mathrm{FR}) \tag{C.9}
$$

We did not explicitly show how the FR changeable terms transform under field redefinitions, but we checked that in fact all coordinate changes are independent, so that we can bring the coordinates of these terms to arbitrary values by choosing an appropriate field redefinition.

As for the most general DUY deformation at this order, it reads:

Superstructure 0: #derivatives: 0, #Fs: 4

Structure 0: Loops: 4

$$\begin{aligned}
d^5_{0,0,0} &: F_{\alpha_1\bar\alpha_2}F_{\alpha_2\bar\alpha_3}F_{\alpha_3\bar\alpha_4}F_{\alpha_4\bar\alpha_1}\\
d^5_{0,0,1} &: F_{\alpha_1\bar\alpha_2}F_{\alpha_2\bar\alpha_3}F_{\alpha_4\bar\alpha_1}F_{\alpha_3\bar\alpha_4}\\
d^5_{0,0,2} &: F_{\alpha_1\bar\alpha_2}F_{\alpha_3\bar\alpha_4}F_{\alpha_2\bar\alpha_3}F_{\alpha_4\bar\alpha_1}\\
d^5_{0,0,3} &: F_{\alpha_1\bar\alpha_2}F_{\alpha_4\bar\alpha_1}F_{\alpha_2\bar\alpha_3}F_{\alpha_3\bar\alpha_4}\\
d^5_{0,0,4} &: F_{\alpha_1\bar\alpha_2}F_{\alpha_3\bar\alpha_4}F_{\alpha_4\bar\alpha_1}F_{\alpha_2\bar\alpha_3}\\
d^5_{0,0,5} &: F_{\alpha_1\bar\alpha_2}F_{\alpha_4\bar\alpha_1}F_{\alpha_3\bar\alpha_4}F_{\alpha_2\bar\alpha_3}
\end{aligned} \tag{C.10}$$

Structure 1: Loops: 2 2

$$\begin{aligned}
d^5_{0,1,0} &: F_{\alpha_1\bar\alpha_2}F_{\alpha_2\bar\alpha_1}F_{\alpha_3\bar\alpha_4}F_{\alpha_4\bar\alpha_3}\\
d^5_{0,1,1} &: F_{\alpha_1\bar\alpha_2}F_{\alpha_3\bar\alpha_4}F_{\alpha_2\bar\alpha_1}F_{\alpha_4\bar\alpha_3}\\
d^5_{0,1,2} &: F_{\alpha_1\bar\alpha_2}F_{\alpha_3\bar\alpha_4}F_{\alpha_4\bar\alpha_3}F_{\alpha_2\bar\alpha_1}
\end{aligned} \tag{C.11}$$

Superstructure 1: #derivatives: 2, #Fs: 3

Structure 0: Chains: 0 Loops: 3

$$\begin{aligned}
d^5_{1,0,12} &: (D_{\alpha_1}F_{\alpha_2\bar\alpha_3})\,(D_{\bar\alpha_1}F_{\alpha_3\bar\alpha_4})\,F_{\alpha_4\bar\alpha_2}\\
d^5_{1,0,13} &: (D_{\alpha_1}F_{\alpha_2\bar\alpha_3})\,F_{\alpha_4\bar\alpha_2}\,(D_{\bar\alpha_1}F_{\alpha_3\bar\alpha_4})\\
d^5_{1,0,14} &: (D_{\bar\alpha_1}F_{\alpha_2\bar\alpha_3})\,(D_{\alpha_1}F_{\alpha_4\bar\alpha_2})\,F_{\alpha_3\bar\alpha_4}\\
d^5_{1,0,15} &: F_{\alpha_2\bar\alpha_3}\,(D_{\alpha_1}F_{\alpha_3\bar\alpha_4})\,(D_{\bar\alpha_1}F_{\alpha_4\bar\alpha_2})\\
d^5_{1,0,16} &: (D_{\bar\alpha_1}F_{\alpha_2\bar\alpha_3})\,F_{\alpha_3\bar\alpha_4}\,(D_{\alpha_1}F_{\alpha_4\bar\alpha_2})\\
d^5_{1,0,17} &: F_{\alpha_2\bar\alpha_3}\,(D_{\bar\alpha_1}F_{\alpha_4\bar\alpha_2})\,(D_{\alpha_1}F_{\alpha_3\bar\alpha_4})\\
d^5_{1,0,18} &: (D_{\bar\alpha_1}F_{\alpha_2\bar\alpha_3})\,(D_{\alpha_1}F_{\alpha_3\bar\alpha_4})\,F_{\alpha_4\bar\alpha_2}\\
d^5_{1,0,19} &: (D_{\bar\alpha_1}F_{\alpha_2\bar\alpha_3})\,F_{\alpha_4\bar\alpha_2}\,(D_{\alpha_1}F_{\alpha_3\bar\alpha_4})\\
d^5_{1,0,20} &: (D_{\alpha_1}F_{\alpha_2\bar\alpha_3})\,(D_{\bar\alpha_1}F_{\alpha_4\bar\alpha_2})\,F_{\alpha_3\bar\alpha_4}\\
d^5_{1,0,21} &: F_{\alpha_2\bar\alpha_3}\,(D_{\bar\alpha_1}F_{\alpha_3\bar\alpha_4})\,(D_{\alpha_1}F_{\alpha_4\bar\alpha_2})\\
d^5_{1,0,22} &: (D_{\alpha_1}F_{\alpha_2\bar\alpha_3})\,F_{\alpha_3\bar\alpha_4}\,(D_{\bar\alpha_1}F_{\alpha_4\bar\alpha_2})\\
d^5_{1,0,23} &: F_{\alpha_2\bar\alpha_3}\,(D_{\alpha_1}F_{\alpha_4\bar\alpha_2})\,(D_{\bar\alpha_1}F_{\alpha_3\bar\alpha_4})
\end{aligned} \tag{C.12}$$

Structure 1: Chains: 1 Loops: 2

$$\begin{aligned}
d^5_{1,1,0} &: F_{\alpha_2\bar\alpha_1}\,(D_{\bar\alpha_2}D_{\alpha_1}F_{\alpha_3\bar\alpha_4})\,F_{\alpha_4\bar\alpha_3}\\
d^5_{1,1,1} &: F_{\alpha_2\bar\alpha_1}F_{\alpha_3\bar\alpha_4}\,(D_{\bar\alpha_2}D_{\alpha_1}F_{\alpha_4\bar\alpha_3})\\
d^5_{1,1,8} &: (D_{\alpha_2}D_{\bar\alpha_1}F_{\alpha_3\bar\alpha_4})\,F_{\alpha_1\bar\alpha_2}F_{\alpha_4\bar\alpha_3}
\end{aligned} \tag{C.13}$$

Superstructure 2: #derivatives: 4, #Fs: 2

Structure 0: Chains: 0 0 Loops: 2

$$\begin{aligned}
&d^5_{2,0,52} : (D_{\alpha_2} D_{\alpha_1} F_{\alpha_3\bar{\alpha}_4}) (D_{\bar{\alpha}_2} D_{\bar{\alpha}_1} F_{\alpha_4\bar{\alpha}_3}) \\
&d^5_{2,0,53} : (D_{\bar{\alpha}_2} D_{\bar{\alpha}_1} F_{\alpha_3\bar{\alpha}_4}) (D_{\alpha_2} D_{\alpha_1} F_{\alpha_4\bar{\alpha}_3}) \\
&d^5_{2,0,54} : (D_{\alpha_2} D_{\bar{\alpha}_1} F_{\alpha_3\bar{\alpha}_4}) (D_{\bar{\alpha}_2} D_{\alpha_1} F_{\alpha_4\bar{\alpha}_3})
\end{aligned} \tag{C.14}$$

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